

Hoping for the best while preparing for the worst in the face of uncertainty: a new type of incomplete preferences*

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Abstract

We propose and axiomatize a new model of incomplete preferences under uncertainty, which we call *hope-and-prepare preferences*. An act is considered more desirable than another when, and only when, both an optimistic evaluation, computed as the welfare level attained in a best-case scenario, and a pessimistic one, computed as the welfare level attained in a worst-case scenario, rank the former above the latter. Our comparison criterion involves multiple priors, as best and worst cases are determined among sets of probability distributions. We make the case that, compared to existing incomplete criteria under ambiguity, hope-and-prepare preferences address the trade-off between conviction and decisiveness in a new way, which is more favorable to decisiveness. We characterize a completion of an incomplete hope-and-prepare preference relation admitting an (asymmetric) α -maxmin expected utility representation, in which α is unique.

Keywords: Decision theory; incomplete preference; dual-selves; α -maxmin expected utility; non-obvious manipulability.

JEL classification: D01; D81; D90

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1 Introduction

“Hoping for the best, prepared for the worst, and unsurprised by anything in between.”

- Maya Angelou, *I Know Why the Caged Bird Sings*.

The complexity of economic decisions is likely to result in agents’ inability or unwillingness to decide over the uncertain options they are supposed to compare. In this regard, the restrictiveness of the assumption that individual preferences be complete was early acknowledged,¹ and was recently highlighted by empirical studies.² We propose and characterize a new *incomplete* decision criterion according to which, in the face of Knightian uncertainty (Knight (1921)), agents *both hope for the best and prepare for the worst*.

We study preferences over acts $f : S \rightarrow X$, which are mappings from states of the world to outcomes, and we introduce and axiomatize preferences $\succ$ admitting the following representation:

$$f \succ g \iff \begin{cases} \min_{p \in C} \int u(f) dp > \min_{p \in C} \int u(g) dp \\ \max_{p \in D} \int u(f) dp > \max_{p \in D} \int u(g) dp \end{cases}, \quad (1)$$

where u is a numerical representation of preferences over outcomes, and C and D are sets of probability distributions over the states, interpreted as sets of different *scenarios*.³ Thus, a decision maker (DM) following such a criterion ranks an act f above an act g if and only if f provides a higher expected utility than g in the worst-case scenario in C as well as in the best-case scenario in D .

Our criterion is based on the conjunction of an optimistic (or ambiguity-seeking) assessment and of a pessimistic (or ambiguity-averse) assessment.⁴ We then interpret a DM with such a preference as hoping for the best scenario to realize, while also preparing for the worst one to happen, when evaluating each option: we thus refer to a preference relation admitting such a representation as a *hope-and-prepare preference*. As a brief illustration, think

¹For instance, Aumann (1962) wrote: “Of all the axioms of utility theory, the completeness axiom is perhaps the most questionable. Like others of the axioms, it is inaccurate as a description of real life; but unlike them, we find it hard to accept even from a normative viewpoint.” Schmeidler (1989), commenting on his characterization of the maxmin criterion, depicted the completeness axiom as “the most restrictive and demanding assumption.”

²See Cettolin and Riedl (2019), Nielsen and Rigotti (2024).

³The function $u : X \rightarrow \mathbb{R}$ is non-constant, affine and unique up to positive affine transformation. The sets C and D are unique, non-disjoint, compact and convex.

⁴ C and D being non-disjoint, the expected utility in the best-case scenario is higher.

of a company considering launching a new product. Typically, such a dual policy of decision making would favor investment or production strategies that present promising profit opportunities, in case the product captures an important market share, *and* a substantial safeguard, in case the product does not.

We shall give special attention in our analysis to the *concordant case* in which $C = D$ —for which we also provide an axiomatization. Acts are then evaluated according to *the interval of all expected utility levels that they induce across all possible scenarios*. More precisely, an act f is preferred to an act g if and only if *any expected utility level that is attainable from g but not from f is below any expected utility level that is attainable from f , and there exists at least one level that is indeed attainable from g but not from f* . This intuitive criterion for comparing *ranges* of expected utility levels works as a strict version of *the strong set order*, which is, arguably, the most common way to compare intervals.

Importantly, hope-and-prepare preferences treat the optimistic and the pessimistic assessments symmetrically: therefore, they do not systematically display a particular attitude toward ambiguity, which is consistent with extensive empirical evidence (see [Trautmann and van de Kuilen \(2015\)](#) for a survey).

The conjunction of a best-case evaluation and of a worst-case evaluation at play in our criterion is akin to the one at play in the notion of *obvious manipulation* ([Trojan and Morrill \(2020\)](#)), defined for revelation games in which the uncertainty faced by an agent concerns others’ messages. Accordingly, the significant practical relevance of the notion of obvious manipulation provides support for our criterion within uncertain strategic environments. This notion gives an explanation, for instance, of untruthful reporting strategies that have been consistently observed in the *Immediate Acceptance mechanism*, used to match students with schools.⁵

The scope for applications of our criterion goes beyond strategic interactions. The idea that *both* worst-case and best-case scenarios serve as reference points is recognized for various social and economic domains where ambiguity is present. In this regard, let us simply mention the evaluation of financial assets ([Bossaerts et al. \(2010\)](#), [Schröder \(2011\)](#), [Ahn et al. \(2014\)](#)), or the evaluation of different medical treatments by physicians and patients ([Back et al. \(2003\)](#), [Taylor et al. \(2017\)](#)); we discuss a third example in more detail.

It is not unusual for practitioners, reporters or fans to evaluate “young prospects” participating in the annual *Draft* in North-American sports leagues —we take the example of the

⁵See [Pathak and Sönmez \(2008\)](#) and [Dur et al. \(2018\)](#).

National Basketball Association league (NBA)— according to “ceiling and floor scenarios.”⁶ This can be formulated in our framework. There is potentially a myriad of parameters that the agent considers relevant for the evaluation of prospects: a state is a particular configuration of parameters.⁷ In this complex environment, the agent faces ambiguity and must compare prospects on the basis of a set C of probability distributions over configurations of parameters. Loosely speaking, each player is identified with an act f , indicating their overall performance in each state, which is then evaluated according to a utility function u , and, for every scenario $p \in C$, the agent can compute the expectation of $u(f)$ according to p . Then, the incompleteness of a criterion such as ours reflects the *necessity to have sufficient conviction* when declaring that a player is more promising than an other one. On the other hand, a criterion should not be *too incomplete*; let us illustrate this point by comparing our criterion to two alternative ones. Given u and C , the agent could require, for a “player f ” to be declared more promising than a “player g ”, that, for each scenario in C , the expected utility associated with g be lower than the expected utility level associated with f (Bewley (2002)). One could even require that any expected utility level attainable from g be lower than any expected utility level attainable from f (Echenique et al. (2022))—that the “ceiling” of g be lower than the “floor” of f . Both of these conditions are stronger than condition (1), expressing a more demanding notion of sufficient conviction. However, it may very well be the case that only “generational talents” such as Victor Wembanyama,⁸ (who was present in the 2023 Draft) be distinguished from other players on the basis of these more conservative criteria, and that for rather homogeneous cohorts such as the 2024 cohort, the agent fail to rank any player above an other one.⁹ In practical terms, according to our criterion, a “player f ” is declared more promising than a “player g ” if and only if anything that g could achieve and that f could not is considered worse than anything f could achieve. With hope-and-prepare preferences, which, in this case, compare players on the basis of the associated ranges of expected utility, in a way that is reminiscent of the strong set order, the *trade-off between decisiveness and conviction* is addressed in a way that is more favorable to

⁶See, for example, James Hansen, “*What makes an NBA Draft prospect high ceiling or high floor?*”, SLC Dunk, June 2023, and Kyle Boone, “*NBA Draft 2024 ceiling and floor scenarios: The best or worst case projections for five top prospects*”, CBS Sports, June 2024. We note that the use of the expressions “ceiling” and “floor” suggests that any case lying “in between” is considered possible.

⁷A state may thus encompass the rosters of coaches and players, at the beginning of the season and after the winter “trade” period, of each franchise, their financial capacities, the performance of players already in the league, the progression of each of these prospects, the approach to officiating favored by the league’s executives, etc.

⁸See, for example, Sam Harris, “*Why ‘alien’ Wembanyama is France’s next big thing - literally*”, BBC Sports, July 2024.

⁹See, for example, Adam Finkelstein, “*No stars have revealed themselves in the 2024 NBA Draft, but history tells us they’re hiding in plain sight*”, CBS Sports, June 23 2024.

decisiveness.

The original axiom involved in our characterization is interpreted along this line: Axiom 6, provides a relatively weak sufficient condition for *comparability*. Our axiomatization maintains the assumption that preferences are complete over constant acts, deemed as the simplest ones. According to Axiom 6, if *i*) the DM cannot compare f to the constant x , while she declares x more desirable than g and, on the other hand, *ii*) she cannot compare g to the constant act y , while she declares f more desirable than y , then she declares f more desirable than g . Thus, *two specific aligned pieces of evidence are enough* to conclude that an act is better than another, and Axiom 6 may be seen as formulating a minimal departure from the completeness of a standard expected utility preference relation.

Furthermore, in order to account for typical situations in which agents *have to* choose between two options, even if they lack conviction to express a clear preference between them in the first place, we study the completion of hope-and-prepare preferences.¹⁰ We demonstrate that the *invariant biseparable complete extension* of a hope-and-prepare preference admits an *asymmetric*¹¹ α -maxmin expected utility (α -MEU) representation —and a standard α -maxmin representation if the hope-and-prepare preference is concordant. Notably, the asymmetric α -MEU retains much of the tractability of the standard α -MEU —which is beneficial for applications— while remaining flexible enough to accommodate mixed ambiguity attitudes (Chandrasekher et al. (2022)).¹² Importantly, in the representation we obtain, *the weight α does not depend on the considered acts, and is unique whenever the extended hope-and-prepare preference is incomplete*.

Finally, answering two natural questions of comparative statics that emerge from the proposition of a new type of incomplete preference under ambiguity, we compare the degree of incompleteness of our criterion to that of Bewley preferences (Bewley (2002)) and of twofold preferences (Echenique et al. (2022)), and we provide a way to compare the ambiguity attitudes of two hope-and-prepare preferences.

Our paper is organized as follows: we define the formal framework and introduce our criterion in Section 2. In Section 3, we give the main representation result and explore the case in which the sets of scenarios used in the two assessments are equal. In Section 4, we investigate the completion of our criterion. Section 5 is dedicated to the comparative statics questions mentioned above. Section 6 provides an illustrative comparison of concordant

¹⁰From a theoretical point of view, studying a completion of an incomplete preference relation enables to use standard mathematical tools, for example for utility maximization and welfare analysis.

¹¹“Asymmetric” refers to the fact that best and worst cases may be taken on different sets of scenarios.

¹²Specifically, it captures ambiguity-averse behavior for *large/moderate-likelihood* events, ambiguity-seeking behavior for *small-likelihood* events, and *source-dependent* ambiguity attitudes (Chandrasekher et al. (2022)).

hope-and-prepare preferences to Bewley preferences, in the context of the aggregation of opinions of experts. The conclusions are presented in Section 7. All proofs can be found in the appendix.

1.1 Related literature

A DM hoping for the best while also preparing for the worst responds to uncertainty by combining opposite ambiguity attitudes. In this perspective, one may interpret a DM with a hope-and-prepare preference as *requiring that her optimistic (ambiguity loving) self and her pessimistic (ambiguity averse) self be unanimous for her to rank some act above another one*. The idea that the DM consists of multiple (strategic) selves appears frequently in behavioral economics, in particular in models of dynamic choice or choice within risky environments.¹³ In recent works, [Chandrasekher et al. \(2022\)](#) and [Xia \(2020\)](#) provided axiomatizations for preferences involving two selves, called by the former *dual-self expected utility*. Their representation differs from ours in that the agent’s final decision is to be interpreted as the result of a specific *leader-follower game* between an optimistic self and a pessimistic self: basically, in order to evaluate an act, the optimistic self moves first by choosing a collection of beliefs that maximizes the evaluation of the pessimistic self, who chooses the belief that minimizes the expected utility of the act among the set chosen by the optimistic self. In contrast, in our representation, the decision is induced by a requirement of unanimity imposed by the agent herself on the assessments of her two selves.¹⁴

Our representation is also motivated by the concept of obvious manipulation proposed in the context of mechanism design by [Troyan and Morrill \(2020\)](#). A revelation mechanism is said to be *non-obviously manipulable* if, for any agent and any potential untruthful report from her, revealing her own type leads to a more desirable outcome in both of the following cases: when the others’ reports are the most favourable to her, and when they are the least favourable. In our model, in the same spirit, an option —such as an untruthful report in the mechanism design context— is abandoned for an alternative only if this alternative leads to preferred outcomes in both the best and the worst scenarios among given sets of probability measures. The relation of our contribution to the concept of non-obvious manipulation mirrors that of [Echenique et al. \(2022\)](#) to the concept of obvious dominance,

¹³[Thaler and Shefrin \(1981\)](#), [Bénabou and Pycia \(2002\)](#), [Fudenberg and Levine \(2006\)](#), [Brocas and Carrillo \(2008\)](#).

¹⁴At a high-level, the difference of our approach with that of “*Preparing for the Worst but Hoping for the Best: Robust (Bayesian) Persuasion*” ([Dworczak and Pavan \(2022\)](#)) is similar to the difference with [Chandrasekher et al. \(2022\)](#): in the criteria studied in both of these papers, one of the (pessimistic or optimistic) evaluations constrains the other. It is not the case with hope-and-prepare preferences in which both evaluations are treated symmetrically.

due to [Li \(2017\)](#). Informally, in a direct mechanism, for each agent, given a belief on the profile of strategic reports of other agents (a scenario), each possible report about her own type (which corresponds to an act), induces an expected payoff. When the set of scenarios according to which all acts are evaluated is the set of distributions whose support is included in the Cartesian product of other agents' type spaces, the act f is preferred to act the g by a *twofold multi-prior preference* if and only if the strategy corresponding to f obviously dominates the strategy corresponding to g , and, on the other hand, f is preferred to g by a *hope-and-prepare preference* if and only if the strategy corresponding to f dominates the strategy corresponding to g in the sense of [Troyan and Morrill \(2020\)](#) —for completeness, we formalize this statement in [Appendix A](#).

Hope-and-prepare preferences define a partial order on acts. Pioneering work by [Aumann \(1962\)](#), [Bewley \(2002\)](#) and [Dubra et al. \(2004\)](#) studied the representation of incomplete preferences under risk and uncertainty. Incomplete preferences in non-deterministic environments have been the object of a growing literature: see, for example, [Nascimento and Riella \(2011\)](#), [Galaabaatar and Karni \(2012\)](#), [Efe et al. \(2012\)](#), [Faro \(2015\)](#), [Minardi and Savochkin \(2015\)](#), [Hill \(2016\)](#), [Karni \(2020\)](#), [Cusumano and Miyashita \(2021\)](#) and [Echenique et al. \(2022\)](#). The closest model of incomplete preference to ours, apart from those studied in [Bewley \(2002\)](#) and [Echenique et al. \(2022\)](#), both compared to ours in the introduction, is introduced in [Nascimento and Riella \(2011\)](#). As a special case of their main result, they study a criterion in which the DM considers several sets of scenarios, in each of which the performance of an act is evaluated according to the worst-case expected utility level. Then, an act is preferred to an other one if and only if it performs better in each set of scenarios. Hope-and-prepare preferences enable to capture a different type of ambiguity attitude, through the consideration of the optimistic assessment. We discuss in more details how our work relates to [Bewley \(2002\)](#), [Nascimento and Riella \(2011\)](#) and [Echenique et al. \(2022\)](#) in the next sections.

In line with Hurwicz's approach for decision making under complete ignorance ([Hurwicz \(1951\)](#)), the α -MEU model was proposed to capture the idea that, under ambiguity, worst and best expected utility levels, over *one* set of probability measures, can serve as sufficient statistics for the DM: she then computes an α -weighted average of these levels ([Marinacci \(2002\)](#), [Kopylov \(2002\)](#), [Ghirardato et al. \(2004\)](#)).¹⁵ Among the recent explorations of (vari-

¹⁵Let us mention two alternatives to the standard α -MEU model. The *geometric* α -MEU model ([Binmore \(2009\)](#)) uses a geometric weighted average. More recently, [Grant et al. \(2020\)](#) introduced and characterized a general aggregation of best-case and worst-case expected utility representations, referred to as *ordinal Hurwicz expected utility*.

ants of) the α -maxmin model,¹⁶ the one of Frick et al. (2022) is particularly important for the way we characterize the *asymmetric* α -maxmin model as representing the completion of hope-and-prepare preferences.¹⁷ In the *objective and subjective rationality* framework proposed by Gilboa et al. (2010), they show that the *invariant biseparable complete extension* of a Bewley preference admits a standard α -MEU representation. We show that the asymmetric α -maxmin representation characterizes the invariant biseparable complete extension of a hope-and-prepare preference relation. It reduces to standard α -maxmin in the case of a concordant hope-and-prepare preference relation. Beyond the fact that we consider the completion of a new type of preferences, our result has three salient features: the representation is asymmetric in general, α does not depend on the considered acts, and is unique.¹⁸

2 Setup and representation

2.1 Model

Our analysis is conducted in the classical framework proposed by Anscombe and Aumann (1963). Uncertainty is modeled through a set S of *states of the world*, endowed with an algebra Σ of subsets of S called *events*, and a non-empty set of consequences X , which is a non-singleton convex subset of a real vector space. A *simple act* is defined as a function $f : S \rightarrow X$ which takes finitely many values and is measurable with respect to Σ ; we denote by $\mathcal{F}$ the set of all simple acts. The *mixture* of two simple acts f and g , for any $\alpha \in [0, 1]$, denoted by $\alpha f + (1 - \alpha)g$, is then defined by setting, for each $s \in S$, $[\alpha f + (1 - \alpha)g](s) = \alpha f(s) + (1 - \alpha)g(s)$. With the usual slight abuse of notation, for all $x \in X$, we use x to denote the constant act defined by $f_x(s) = x$ for all $s \in S$. We use Δ to denote the set of all finitely additive probability distributions on (S, Σ) , endowed with the weak* topology.¹⁹ We refer to a measure $p \in \Delta$ as a *scenario* according to which simple acts are evaluated.²⁰

We consider a DM whose preference is represented by a binary relation $\succ \subseteq \mathcal{F} \times \mathcal{F}$. It is a partial ranking over acts and we use the standard notation $f \succ g$ to denote $(f, g) \in \succ$.

¹⁶Chateauneuf et al. (2007), Eichberger et al. (2011), Gul and Pesendorfer (2015), Frick et al. (2022), Klibanoff et al. (2022), Hartmann (2023), Hill (2023), Chateauneuf et al. (2024).

¹⁷Recall that “asymmetric” refers to the fact that best and worst cases may be taken on different sets of scenarios.

¹⁸More precisely, α is unique whenever the considered hope-and-prepare preference relation is not complete.

¹⁹The set of finitely additive bounded measures on (S, Σ) is the dual of the set of all measurable real-valued bounded functions on (S, Σ) . Thus the weak* topology on Δ is defined according to the following convergence notion: we say that a sequence $\{p_n\}$ of elements of Δ converges to $p \in \Delta$ if for all measurable bounded function $\varphi : S \rightarrow \mathbb{R}$, $\int \varphi dp_n$ converges to $\int \varphi dp$.

²⁰From now on, we refer to simple acts as “acts”.

If $f \succsim g$ and $g \succsim f$, we write $f \asymp g$, and say that f and g are *incomparable*.²¹ We interpret $f \succ g$ as reflecting the fact that the DM considers that f is more desirable than g *with sufficient conviction*. In other words, in each pairwise comparison, one act (g in the previous notation) has the role of a *default* that would be abandoned only if the DM had enough reasons to believe that the alternative performs better.

We denote the set of vectors whose k elements are non-negative by $\mathbb{R}_+^k$, the set of vectors whose k elements are positive by $\mathbb{R}_{++}^k$, for any natural number k . For a given set A , $|A|$ denotes the cardinality of A .

2.2 Hope-and-prepare preferences

2.2.1 Definition

Our representation involves multiple priors:²² the DM has a set of relevant beliefs according to which she evaluates acts.

Definition 1. A binary relation $\succ$ is a *hope-and-prepare* preference if

$$f \succ g \iff \begin{cases} \min_{p \in C} \int u(f) dp > \min_{p \in C} \int u(g) dp \\ \max_{p \in D} \int u(f) dp > \max_{p \in D} \int u(g) dp \end{cases},$$

where u is a non-constant affine function defined on X , and C and D are two compact and convex subsets of Δ with $C \cap D \neq \emptyset$.

The representation is *concordant* if $C = D$.

We sometimes write that $\succ$ admits the representation (u, C, D) to refer to the hope-and-prepare representation given in Definition 1. We obtain in our axiomatization the uniqueness up to affine transformation of u , and the uniqueness of C and D . *We sometimes write, then, as a shortcut, that $\succ$ admits the unique representation (u, C, D) .*²³

2.2.2 Discussion

Consider such a preference relation $\succ$, with representation (u, C, D) . An act f is preferred to an act g if and only if f gives a higher expected utility than g when they are evaluated according to their best-case scenario in D , *and* gives a higher expected utility than g

²¹ Accordingly, we say that f and g are *comparable* if either $f \succ g$ or $g \succ f$.

²² Etner et al. (2012) and Gilboa and Marinacci (2016) both provide a review of the ways in which ambiguity, and ambiguity attitudes, have been modeled in order to offer alternatives to the traditional Bayesian framework. Multiple prior models stand out as one of the main lines of research.

²³ As opposed to writing that $\succ$ admits representation (u, C, D) , where u is unique, *up to affine transformation*, and C and D are unique.

when they are evaluated according to their worst-case scenario in C . This conjunction of an optimistic (or ambiguity-seeking) assessment and of a pessimistic (or ambiguity-averse) assessment models a DM hoping for the best scenario to realize, while also preparing for the worst one, when she evaluates each option.

The combination of two such opposite ambiguity attitudes may also be interpreted in the perspective of a DM consisting of two selves: for the DM to consider with sufficiently strong conviction that an act f is more desirable than an act g , it is necessary, and sufficient, that her optimistic self and pessimistic self, be unanimous over the ranking of f and g .²⁴

Let $\succ$ be a hope-and-prepare preference relation with representation (u, C, C) . Then, the DM evaluates any act f in terms of its range $R(f) = \{\int u(f)d\mu : \mu \in C\}$ of possible expected utility levels, which, as C is convex and compact, is a closed interval. Consider an other act $g \in \mathcal{F}$ and suppose that $R(f) = [a, b]$ and $R(g) = [c, d]$. Then f is preferred to g if and only if $a > c$ and $b > d$. This intuitive criterion for comparing ranges of expected utility levels works as an asymmetric version of *the strong set order*, applied to the special case of intervals: f is preferred to g if and only if *any expected utility level that is attainable from g but not from f is below any expected utility level that is attainable from f , and there exists at least one level that is indeed attainable from g but not from f .*

Relation to other incomplete criteria under ambiguity. At this point, it is interesting to describe how the way ranges are compared according to concordant hope-and-prepare preferences can be formally related to the way in which they are compared according to concordant *twofold preferences*, introduced and axiomatized by [Echenique et al. \(2022\)](#).

Definition 2. ([Echenique et al. \(2022\)](#)) A binary relation $\succ$ is a (multi-prior) *twofold preference* if

$$f \succ g \iff \min_{p \in C} \int u(f)dp > \max_{p \in D} \int u(g)dp,$$

where u is a non-constant affine function defined on X , C and D are two compact and convex subsets of Δ with $C \cap D \neq \emptyset$. The representation is said concordant if $C = D$.²⁵

Consider $\succ_{HP}$ a concordant hope-and-prepare preference and $\succ_T$ a concordant twofold preference with the same representing utility function u on X and the same set of scenarios $C \in \Delta$. For any two real intervals I and I' , we say that I *lies above* I' whenever $u > v$ for any $u \in I, v \in I'$. One has $f \succ_T g$ if and only if the interval $R(f)$ lies above the interval $R(g)$, and $f \succ_{HP} g$ if and only if $R(f) \setminus R(g)$ is non-empty and lies above the interval $R(g) \setminus R(f)$, also non-empty.

²⁴In a previous version of this work, hope-and-prepare preferences were called *unanimous dual-self preferences*.

²⁵They obtain the uniqueness, up to affine transformation, of u , and the uniqueness of C and D in their axiomatization.

Accordingly, given a set of scenarios, for a concordant hope-and-prepare preference and a concordant twofold preference, of which the restrictions to outcomes (constant acts) are equal, the former is always *more complete* than the latter, in the sense that it is an order extension of it —a general statement, beyond the case of concordant preferences, is given in Proposition 1. This observation is the basis on which we compared concordant hope-and-prepare preferences to concordant twofold preferences in the NBA example of the introduction, and a similar comparison to Bewley preferences should be made (again, see Proposition 1):

Definition 3. (Bewley (2002)) A binary relation $\succ$ is a (multi-prior) *Bewley preference* if

$$f \succ g \iff \int u(f)dp > \int u(g)dp \text{ for all } p \in C,$$

where u is a non-constant affine function defined on X , C is a non-empty compact and convex subset of Δ .²⁶

As we highlighted in the NBA example, with hope-and-prepare preferences, the trade-off between decisiveness and conviction is addressed in a way that is more favorable to decisiveness, compared to twofold preferences and to Bewley preferences. That is, our criterion still reflects the necessity for DM to have sufficient conviction when declaring an act more desirable than an other one, while it induces more choices. The two original axioms involved in our characterization are interpreted along this line in Section 3.1.1.

A special case of the preferences studied in Nascimento and Riella (2011) is, as ours, defined by the conjunction of different assessments. These preferences are pre orders,²⁷ in contrast to hope-and-preferences, in general. We thus adapt their definition —see Theorem 4 in Nascimento and Riella (2011)— by replacing weak inequalities by strict inequalities.

Definition 4. (Nascimento and Riella (2011)) A binary relation $\succ$ is a *N&R preference* if

$$f \succ g \iff \min_{p \in C} \int u(f)dp > \min_{p \in C} \int u(g)dp \text{ for all } C \in \mathcal{C},$$

where u is a non-constant affine function defined on X , $\mathcal{C}$ is a class of non-empty compact and convex subsets of Δ .

Our approach and criterion differ from those of Nascimento and Riella (2011) in several aspects. In terms of methodology, in contrast to them, we provide an axiomatization directly

²⁶Similarly to the expression we used for our criterion, we will say that the twofold preference $\succ_T$ admits the unique representation (u, C_T, D_T) , and that the Bewley preference $\succ_B$ admits the unique representation (u, C_B) to refer to the fact that u is unique up to affine transformation, and C_T , D_T and C_B are unique.

²⁷That is, reflexive and transitive binary relations.

on the domain of simple acts.²⁸ In addition, in our representation result, the pair of sets of probability measures is unique.²⁹

Furthermore, while the set $\mathcal{C}$ may be infinite in their model—the DM then takes decisions based on the unanimity of an arbitrary, potentially infinite, number of selves—our criterion requires the conjunction of merely two evaluations.

From a behavioral perspective, N&R preferences are based on the unanimity of a collection of MEU representations, reflecting a pessimistic attitude toward ambiguity. However, substantial experimental evidence points to more nuanced patterns of ambiguity attitudes: the same individuals may exhibit ambiguity aversion in some decision problems while displaying ambiguity-seeking behavior in others—see [Trautmann and van de Kuilen \(2015\)](#) for a survey. In contrast to N&R preferences, hope-and-prepare preferences encompass, in addition to a pessimistic perspective, an optimistic one, and therefore, do not exhibit a systematic attitude toward ambiguity. In line with this feature of a hope-and-prepare preference relation, which is incomplete, we show that an invariant biseparable completion of such a preference admits an asymmetric α -MEU representation, which can accommodate a mixture of different ambiguity attitudes (see Section 4).

Finally, we emphasized the importance of the special case of concordant hope-and-prepare preferences, defined by a very intuitive comparison of ranges of expected utility: this has no counterpart in the N&R model.

Non-concordant hope-and-prepare preferences. We note that in the case of a non-concordant representation, the two sets of beliefs may be obtained as the DM distorts the probability measures of a single set, possibly informed by expert sources. The DM may distort probabilities differently when adopting an optimistic versus a pessimistic perspective, as illustrated in the following example.

Example. Consider an NBA fan who may bet on the next game involving her favorite team, team A, which, sadly, happens not to be a “500 team” in the current season.³⁰ The opponent, team B, who has a better record, is more likely to win the game. The set of states of the world is the set of possible scores for the teams, $S = [70, 140] \times [70, 140]$, of generic element (s_A, s_B) . The set of events Σ is the set of all subsets of S . The DM may take a bet on the subset $S_{\{A \text{ wins}\}} = \{(s_A, s_B) \in S \mid s_A \geq s_B\}$ —either A wins, or the teams have equal

²⁸Their result is obtained on the set of lotteries on simple acts.

²⁹In their representation, $\mathcal{C}$ is not unique. The authors obtain the uniqueness of the closure of the convex hull of $\mathcal{C}$, where the set of subsets of the simplex is endowed with the Hausdorff topology.

³⁰This expression is used to denote teams that have lost the majority of their games during the current season.

scores before the overtime starts— on the basis of several probability measures on (S, Σ) reflecting the opinions of different experts or brokers. Let C be (the convex hull of) this set of probability measures, and assume that the value associated with $S_{\{A \text{ wins}\}}$ across measures in C browses $[\frac{1}{3}, \frac{1}{2}]$.

The DM has a utility function over money $u : \mathbb{R} \rightarrow \mathbb{R}$, $u(x) = \frac{1}{2}x$, and must compare, in particular, the two following bets, or acts: for all $s \in S$,

$$\begin{aligned} f(s) &= 50\$ \text{ if } s \in S_{\{A \text{ wins}\}}, \\ f(s) &= -30\$ \text{ otherwise;} \\ g(s) &= 34\$ \text{ if } s \in S_{\{A \text{ wins}\}}^c, \\ g(s) &= -30\$ \text{ otherwise,} \end{aligned}$$

where $S_{\{A \text{ wins}\}}^c$ is the complement of $S_{\{A \text{ wins}\}}$. The act g amounts to betting 30 dollars *against* team A.

If the DM is consequentialist in the sense that she evaluates acts exclusively according to the payoff they may induce, then C appears as the natural set of scenarios to use: according to the hope-and-prepare preference relation $\succ$, with representation (u, C, C) , $g \succ f$.³¹

On the other hand, non-concordant hope-and-prepare preferences can capture the non-consequentialist reluctance of the DM to bet against her favorite team. This type of non-consequentialism has long been documented in the psychology literature on “valence”, has been established in the sport context by several recent empirical studies (Morewedge et al. (2018), Kossuth et al. (2020), Donkor et al. (2023)), and has motivated the axiomatic work of Adam et al. (2024). In the context of this example, starting from C , the DM, while hoping for the best, may, for instance, distort probability measures giving too high a value to states—which she dislikes—in which team A loses by too high a margin. Let D be this set of distorted probability measures, and assume that the value associated with $S_{\{A \text{ wins}\}}$ across measures in D browses $[\frac{1}{3} + 0.02, \frac{1}{2} + 0.02]$. Assume that, on the contrary, when she performs a pessimistic assessment, the DM does not depart from the scenarios given by the experts. According to the hope-and-prepare

³¹The minimal and maximal expected utility level induced by f is $25 \times \frac{1}{3} - 15 \times \frac{2}{3} \approx -1.67$, and the maximal one is $25 \times \frac{1}{2} - 15 \times \frac{1}{2} = 5$. The minimal expected utility level induced by g is $17 \times \frac{1}{2} - 15 \times \frac{1}{2} = 1$, and the maximal one is $17 \times \frac{2}{3} - 15 \times \frac{1}{3} \approx 6.33$.

preference relation $\succ'$, with representation (u, C, D) , $f \bowtie' g$.³² Thus, in contrast to the concordant case above, given these potential rewards, the DM does not prefer betting 30\$ against her favorite team compared to betting 30\$ in favor of it.³³ Of course, sufficiently high monetary rewards in case team B wins can compensate the reluctance of the DM to bet against her favorite team when her preference is $\succ'$, as can be checked by replacing 34\$ by 42\$ in the definition of g .

Best and worst outcomes in the lab. The phenomenon that, in general, individuals overweight extreme outcomes, when they face known probabilities, as well as when they face ambiguous ones, is well documented. A large body of experimental studies —surveyed in [Trautmann and van de Kuilen \(2015\)](#)— have highlighted the presence of such *insensitivity* among student subjects, and subsequent research, conducted among the general population, has confirmed its importance ([Dimmock et al. \(2016\)](#), [Watanabe and Fujimi \(2024\)](#)).³⁴ The idea that options should be compared *only* based on extreme outcomes corresponds to complete insensitivity.³⁵

³²The minimal expected utility level induced by f over C is $25 \times \frac{1}{3} - 15 \times \frac{2}{3} \approx -1.67$, and the maximal one over D is $25 \times (\frac{1}{2} + 0.02) - 15 \times (\frac{1}{2} - 0.02) = 5.8$. The minimal expected utility level induced by g over C is $17 \times \frac{1}{2} - 15 \times \frac{1}{2} = 1$, and the maximal one over D is $17 \times (\frac{2}{3} - 0.02) - 15 \times (\frac{1}{3} + 0.02) \approx 5.69$.

³³Note that the incomparability of f and g would not be obtained with the concordant preference defined using the convex hull of C and D . Let $\tilde{C}$ denote this convex hull. Then, the value associated with $S_{\{A \text{ wins}\}}$ across measures in $\tilde{C}$ browses $[\frac{1}{3}, \frac{1}{2} + 0.02]$. The minimal and maximal expected utility levels induced by f over $\tilde{C}$ are -1.67 and 5.8 , respectively, while the minimal and maximal expected utility levels induced by g are 0.36 and 6.33 , respectively. Thus, according to the hope-and-prepare preference relation $\tilde{\succ}$, with representation $(u, \tilde{C}, \tilde{C})$, $g \tilde{\succ} f$.

³⁴The reader can find a thorough account of the numerous studies pointing out insensitivity in the Online Appendix D.2 of [Baillon et al. \(2025\)](#).

³⁵Our analysis models ambiguity rather than risk, as the DM has multiple priors. Despite this difference, the psychological mechanisms behind the two approaches are closely related, and findings obtained for outcomes are relevant for models involving multiple priors. The connection between them can be made explicit as follows —we are grateful to Peter Wakker for pointing out this connection. *For a given act*, the DM can associate each prior with the induced expected utility (EU) of the act. Assume that the DM computes an *EU-certainty-equivalent* (EU-CE) of each prior, given this act, that is, an outcome yielding a utility level equal to the EU level induced by the prior. Each act is now associated with a set of certainty-equivalents, one for each prior, and acts are compared on the basis of these sets. In other words, each act induces a *mapping from priors to their EU-CE*. And the highest and lowest expected utility levels according to which the act is evaluated with hope-and-prepare preferences are equal to the highest and lowest values taken by the corresponding mapping.

3 Representation results

3.1 Characterization of hope-and-prepare preferences

3.1.1 Axioms

We now proceed to an axiomatic characterization of hope-and-prepare preferences based on the following seven axioms. Axioms 1, 2, 3 and 5 express common requirements, Axiom 4 was proposed in Echenique et al. (2022), 6 is original to this work.

Axiom 1. Relation $\succ$ is asymmetric and transitive, and the restriction of $\succ$ to X is non-trivial and negatively transitive.³⁶

Axiom 2. For all triple $(f, g, h) \in \mathcal{F}^3$, the sets $\{\alpha \in [0, 1] : \alpha f + (1 - \alpha)g \succ h\}$ and $\{\alpha \in [0, 1] : h \succ \alpha f + (1 - \alpha)g\}$ are open.

Axiom 3. For all $f, g \in \mathcal{F}$, $x \in X$, and $\alpha \in (0, 1)$, $f \succ g$ if and only if $\alpha f + (1 - \alpha)x \succ \alpha g + (1 - \alpha)x$.

The interpretation of the assumptions in Axiom 1 is well-known. In particular, on X , $\succ$ is the asymmetric part of a complete and transitive relation. Axiom 2 is the standard Archimedean continuity condition adopted in models of decision under uncertainty. Axiom 3 is the independence axiom proposed by Gilboa and Schmeidler (1989) in their seminal paper as a weakening of the independence axiom at play in the characterization of subjective expected utility.

Axiom 4. For all $x \in X$, the sets $\{f \in \mathcal{F} : f \succ x\}$ and $\{f \in \mathcal{F} : x \succ f\}$ are convex.

Axiom 4 is identical to Axiom 4 in Echenique et al. (2022), which states that comparisons to a given constant act should not be sensitive to hedging. Recall that since $\succ$ is asymmetric and incomplete, in general, $\{f \in \mathcal{F} : f \succ x\}$ is interpreted as the set of acts for which the DM has sufficient conviction to consider them more desirable than the constant act x . The convexity of $\{f \in \mathcal{F} : f \succ x\}$ is interpreted in terms of uncertainty aversion: an act obtained through hedging between two acts that provide sufficient evidence to be declared more desirable than the constant act x is also considered more desirable than x with sufficient conviction. The convexity of $\{f \in \mathcal{F} : x \succ f\}$ is interpreted in terms of preference for uncertainty: when the DM has sufficient conviction to declare two uncertain acts less desirable than the constant act x , then the DM also considers with sufficient conviction that an act obtained through hedging between the two is less desirable than x .

³⁶Negative transitivity of $\succ$ means that for all $x, y, z \in X$, if $x \not\succ y$ and $y \not\succ z$ then $x \not\succ z$.

Axiom 5. For all $f, g \in \mathcal{F}$, if $f(s) \succ g(s)$ for all $s \in S$, then $f \succ g$.

We impose a relatively weak sufficient condition for comparability in Axiom 6.

Axiom 6. For all $f, g \in \mathcal{F}$, and for all $x, y \in X$, if $f \bowtie x$, $x \succ g$, $g \bowtie y$ and $f \succ y$ then $f \succ g$.

While the DM cannot compare f to the constant x , she declares x more desirable than g . On the other hand, while she cannot compare g to the constant act y , she declares f more desirable than y . Axiom 6 implies that *in the presence of such consonant conclusions as to the comparison of f and g , the DM considers f , with sufficient conviction, more desirable than g .*

As we already highlighted, given the complexity involved in the evaluation of uncertain acts, constant acts, which are the simplest acts, involving no ambiguity, are likely to be used as comparison devices. A straightforward way to use them in comparing two acts, when preferences may be incomplete, then consists in looking for a constant act that is incomparable to one of them and comparable to the other one. Each such constant act then provides *a piece of evidence* as to the comparison between the two uncertain acts —the question is then to determine what are sufficient pieces of evidence.

According to Axiom 6, two consonant pieces of evidence suffice: if there is a constant act x that is incomparable to f and more desirable than g , and a constant act y that is incomparable to g and less desirable than f , then both pieces of evidence favor f , and the DM declares f more desirable than g .

There is a sense in which Axiom 6 expresses, for an asymmetric and incomplete preference relation, a minimal departure from the completeness of weak orders for which all acts admit a *certainty equivalent*.³⁷ For these weak orders, one piece of evidence is sufficient: if $f \in \mathcal{F}$ has a certainty equivalent $x \in X$ and x is strictly preferred to $g \in \mathcal{F}$, then f is strictly preferred to g . For an asymmetric and incomplete preference relation, Axiom 6 involves no more than one piece of evidence based on a constant act incomparable to f and one piece of evidence based on a constant act incomparable to g .

Axiom 6 is violated by twofold preferences, Bewley preferences and N&R preferences in general. Through the satisfaction of Axiom 6, in particular, hope-and-prepare preferences address the trade-off between decisiveness and conviction in a new way.

We sometimes refer to the classical Axioms 2, 3 and 5 as continuity, certainty independence and monotonicity.

³⁷That is, binary relations $\succeq$ which are reflexive, transitive and complete, such that, for all $f \in \mathcal{F}$, there is $x \in X$ such that $f \sim x$.

3.1.2 First characterization theorem

Theorem 1. A binary relation $\succ$ satisfies Axioms 1-6 if and only if there exist

- a non-constant affine function $u : X \rightarrow \mathbb{R}$, unique up to positive affine transformation,
- a unique pair (C, D) of non-disjoint convex compact subsets of Δ ,

such that, for all $f, g \in \mathcal{F}$,

$$f \succ g \Leftrightarrow \begin{cases} \min_{p \in C} \int u(f) dp > \min_{p \in C} \int u(g) dp \\ \max_{p \in D} \int u(f) dp > \max_{p \in D} \int u(g) dp \end{cases},$$

that is, $\succ$ admits the hope-and-prepare representation (u, C, D) , where C and D are unique, and u is unique up to positive affine transformation.

In the proof, we define two relations (pre orders) on the bases of $\succ$: for all $f, g \in \mathcal{F}$,

$$\begin{aligned} f \succsim_p g &\iff \text{for all } x \in X, g \succ x \text{ implies } f \succ x, \\ f \succsim_o g &\iff \text{for all } x \in X, x \succ f \text{ implies } x \succ g. \end{aligned}$$

These relations capture the *pessimistic* and *optimistic* assessments underlying $\succsim$: $f \succsim_p g$ whenever f passes every threshold that g passes, and $f \succsim_o g$ whenever every constant act that is at least as desirable as f is also at least as desirable as g .

3.2 Characterization of concordant hope-and-prepare preferences

3.2.1 Axioms

We now identify conditions under which $C = D$ in the hope-and-prepare representation. Under certainty independence (Axiom 3), comparisons between acts are preserved under mixtures with constant acts. The following two axioms impose that comparisons between an act and a constant act be preserved under mixtures with *any* act.

Axiom 7. For all $f \in \mathcal{F}$, $x \in X$, $h \in \mathcal{F}$, and $\alpha \in (0, 1]$, $f \succ x$ implies $\alpha f + (1 - \alpha)h \succ \alpha x + (1 - \alpha)h$.

Axiom 8. For all $f \in \mathcal{F}$, $x \in X$, $h \in \mathcal{F}$, and $\alpha \in (0, 1]$, $x \succ f$ implies $\alpha x + (1 - \alpha)h \succ \alpha f + (1 - \alpha)h$.

Axiom 7 states that being more desirable than a constant act reflects an unambiguously favorable assessment: it cannot be overturned by mixing with any act. Axiom 8 states symmetrically that being less desirable than a constant act reflects an unambiguously unfavorable assessment.

For a hope-and-prepare preference relation, the two axioms ensure that the pessimistic and optimistic evaluations are computed on the same set of scenarios.³⁸

3.2.2 Second characterization theorem

Theorem 2. *The following statements hold:*

- (i) *A hope-and-prepare preference $\succ$, with unique representation (u, C, D) , satisfies Axiom 7 if and only if $D \subseteq C$.*
- (ii) *A hope-and-prepare preference $\succ$, with unique representation (u, C, D) , satisfies Axiom 8 if and only if $C \subseteq D$.*

In particular, a binary relation $\succ$ satisfies Axioms 1-8 if and only if there exist

- *a non-constant affine function $u : X \rightarrow \mathbb{R}$, unique up to positive affine transformation,*
- *a unique convex compact subset of Δ , denoted C , such that, for all $f, g \in \mathcal{F}$,*

$$f \succ g \iff \begin{cases} \min_{p \in C} \int u(f) dp > \min_{p \in C} \int u(g) dp \\ \max_{p \in C} \int u(f) dp > \max_{p \in C} \int u(g) dp \end{cases}.$$

When $\succ$ admits a concordant representation, acts are evaluated according to the interval of all expected utility levels that they induce across scenarios in a given set. An act f is preferred to g if and only if any expected utility level that is attainable from g but not from f is below any expected utility level that is attainable from f , and there exists at least one such level.

4 Complete extension of hope-and-prepare preferences

In this section, we will explore the extension of hope-and-prepare preferences to complete preferences. We will focus on the *invariant biseparable* complete extension; these define a broad class of complete preferences that nests the majority of preferences studied in the literature.

³⁸We thank an anonymous referee who suggested adopting these two axioms.

We refer to an asymmetric complete and negatively transitive binary relation on $\mathcal{F}$ satisfying Axioms 2, 3 and 5 as *invariant biseparable*.³⁹ When it is, in addition, a weak order, it satisfies the axioms characterizing expected utility, apart from the independence axiom, which is weakened to the certainty independence property introduced in Gilboa and Schmeidler (1989).

Definition 5. A preference relation $\succ$ on $\mathcal{F}$ admits an *asymmetric α -MEU* representation if there exist $\alpha \in [0, 1]$, two non-disjoint compact convex subsets C and D of Δ , and a non-constant affine function $u : X \rightarrow \mathbb{R}$ such that for all $f, g \in \mathcal{F}$,

$$\begin{aligned} f \succ g &\iff \alpha \min_{p \in C} \int u(f) dp + (1 - \alpha) \max_{p \in D} \int u(f) dp \\ &> \alpha \min_{p \in C} \int u(g) dp + (1 - \alpha) \max_{p \in D} \int u(g) dp. \end{aligned}$$

We will refer to such representation as a (u, C, D, α) representation.

Remarkably, Chandrasekher et al. (2022) show that the asymmetric α -MEU, while retaining the tractability property of the standard α -MEU, is flexible enough to accommodate ambiguity-averse for *large/moderate-likelihood* events but ambiguity-seeking for *small-likelihood events* and *source-dependent* ambiguity attitudes.

Standard α -MEU criteria are obtained if $C = D$ in Definition 5, and the following result, as a particular case, characterizes them as invariant bi-separable extensions of concordant hope-and-prepare preferences.

Theorem 3. *The following conditions are equivalent when $\succ$ is a hope-and-prepare preference with unique representation (u, C, D) :*

- (i) $\succ^*$ is an invariant biseparable preference and an extension of $\succ$.
- (ii) $\succ^*$ admits an α -maxmin expected utility representation (u, C, D, α) in which α is unique whenever $\succ$ is not complete.

Beyond the fact that we consider the completion of a new type of preferences, our characterization has three salient feature compared to other inquiries about the α -MEU criterion: the representation is asymmetric in general, α does not depend on the considered acts, and is essentially unique.

³⁹Ghirardato et al. (2004) originally used the expression “invariant biseparable preferences” when studying weak orders. For an asymmetric complete and negatively transitive binary relation $\succ$ on $\mathcal{F}$, as for $\succ_p$ and $\succ_o$ in Section 3, we define $\sim$ by $f \sim g$ if and only if $f \not\succ g$ and $g \not\succ f$, for all $f, g \in \mathcal{F}$, and $\succeq$ by $f \succeq g$ if and only if either $f \succ g$ or $f \sim g$, for all $f, g \in \mathcal{F}$. Then, in the proof of Theorem 3, we show that $\succeq^*$ is an “invariant biseparable preference” in the sense of Ghirardato et al. (2004).

5 Comparison of incomplete criteria

5.1 Degree of incompleteness

We have stated that with hope-and-prepare preferences, in comparison to Bewley preferences and twofold preferences, the trade-off between decisiveness and conviction is addressed in a way that is more favorable to decisiveness. The criterion we use to determine whether a binary relation is more conservative than an other one pertains to their respective *degree of incompleteness*.

Definition 6. Given two preference relations $\succ_1$ and $\succ_2$ on $\mathcal{F}$, we say that $\succ_1$ is more conservative than $\succ_2$ if $\succ_2$ is an extension of $\succ_1$, that is, for all $f, g \in \mathcal{F}$,

$$f \succ_1 g \text{ implies } f \succ_2 g.$$

The next proposition identifies necessary and sufficient conditions under which a hope-and-prepare preference relation is an extension of a Bewley or of a twofold preference relation.

Proposition 1. *Let $\succ_{HP}$ be a hope-and-prepare preference with unique representation (u, C_{HP}, D_{HP}) . Let $\succ_T$ be a twofold multiprior preference with unique representation (u, C_T, D_T) . Let $\succ_B$ be a Bewley preference with unique representation (u, C_B) . Then,*

- (i) *the preference relation $\succ_B$ is more conservative than $\succ_{HP}$ if and only if $C_{HP} \cup D_{HP} \subseteq C_B$;*
- (ii) *the preference relation $\succ_T$ is more conservative than $\succ_{HP}$ if and only if $C_{HP} \subseteq C_T$ and $D_{HP} \subseteq D_T$.*

Remark 1. A direct consequence of this proposition and Proposition 4 in [Echenique et al. \(2022\)](#) is that if $C_{HP} \cup D_{HP} \subseteq C_B \subseteq C_T \cap D_T$, in particular if $C_{HP} = D_{HP} = C_B = C_T = D_T$, then $\succ_T$ is more conservative than $\succ_B$, which is more conservative than $\succ_{HP}$.

5.2 Ambiguity attitudes

We are able to compare ambiguity attitudes displayed by different hope-and-prepare preferences using the classical comparative statics notions of [Ghirardato and Marinacci \(2002\)](#).

Definition 7. Given two preference relations $\succ_1$ and $\succ_2$ on $\mathcal{F}$,

- (i) $\succ_1$ is more ambiguity averse than $\succ_2$ if, for all $f \in \mathcal{F}$ and $x \in X$, $f \succ_1 x$ implies $f \succ_2 x$.

- (ii) $\succ_1$ is more ambiguity loving than $\succ_2$ if, for all $f \in \mathcal{F}$ and $x \in X$, $x \succ_1 f$ implies $x \succ_2 f$.

An agent is more ambiguity averse than an other one if she is less inclined to choose an uncertain act f over a constant act x . On the other hand, an agent is more uncertainty loving than an other one if she is more inclined to stick to an uncertain act f than to switch to a constant act x . The next result characterizes ambiguity attitudes for hope-and-prepare preferences.

Proposition 2. *Let $\succ_1$ and $\succ_2$ be two hope-and-prepare preference relations with unique representation (u, C_1, D_1) and (u, C_2, D_2) , respectively. Then,*

(i) *$\succ_1$ is more ambiguity averse than $\succ_2$ if and only if $C_2 \subseteq C_1$.*

(ii) *$\succ_1$ is more ambiguity loving than $\succ_2$ if and only if $D_2 \subseteq D_1$.*

For a hope-and-prepare representation (u, C, D) , the two sets of priors C and D represent the level of pessimism and optimism related to the DM's ambiguity attitudes. More precisely, the relationship $C_2 \subseteq C_1$ means that, in the worst scenario, the level of welfare attained by the agent if she has preference relation $\succ_1$ is lower than the one attained if she has preference relation $\succ_2$. Similarly, $D_2 \subseteq D_1$ means that, in the best scenario, the level of welfare attained by the agent if she has preference relation $\succ_1$ is higher than the one attained if she has preference relation $\succ_2$.

Based on Proposition 2 (i), by comparing the concordant preference $\succ$ with representation (u, C, C) to the non-concordant preference $\succ_1$ with representation (u, C_1, C) , with $C_1 \subset C$, we can say that $\succ_1$ is more ambiguity averse than it is ambiguity loving. Similarly, the non-concordant representation $\succ_2$ with representation (u, C, D_2) , with $D_2 \subset C$, can be said to be more ambiguity loving than it is ambiguity averse. Then, a DM with concordant preferences is as ambiguity loving as she is ambiguity averse, or, in other words, *her pessimistic evaluation is as pessimistic as her optimistic evaluation is optimistic*.

We end this subsection by briefly discussing the relation between the degree of conservatism of a hope-and-prepare preference relation and the attitude towards ambiguity that it displays. It is easy to see that if $\succ_1$ and $\succ_2$ are hope-and-prepare preferences, and if $\succ_1$ is more conservative than $\succ_2$, then $\succ_1$ is both more ambiguity averse and more ambiguity loving than $\succ_2$. Does the converse statement hold? This question is all the more natural that if $\succ_1$ and $\succ_2$ are twofold preferences, then $\succ_1$ is more conservative than $\succ_2$ if, and only if, $\succ_1$ is more ambiguity averse and more ambiguity loving than $\succ_2$.⁴⁰ An example in Appendix B shows that the answer is negative for hope-and-prepare preferences.

⁴⁰See Corollary 1 in Echenique et al. (2022).

6 Aggregating the opinion of experts with hope-and-prepare preferences

Numerous economic decisions under uncertainty, such as those related to fiscal policy and those addressing climate change, often hinge on the guidance provided by groups of experts, who frequently hold conflicting “opinions.” We propose a simple illustration, in the context of the aggregation of conflicting opinions among experts, in which the fact that the planner’s decisions are taken according to a hope-and-prepare preference relation rather than according to a Bewley one reflects her preference for decisiveness.

Due to the complexity of the issue at hand, the opinions of experts may encompass several probability distributions (scenarios) over payoff-contingent states. Following [Danan et al. \(2016\)](#), we assume that experts have Bewley preferences, which expresses, given a set of plausible scenarios, the need of experts to have a strong conviction in order to report to the planner (the DM) that an option is better than an other one:⁴¹

“[...] a given individual may also consider more than one model to be plausible—or have an imprecise belief. For such an individual, which of two policies yields the highest expected utility may depend on the model considered. When a policy yields a higher expected utility than another one for all plausible models, we say that the individual unambiguously prefers the former policy to the latter. Unambiguous preferences are thus robust to belief imprecision.”

Let $N = \{1, 2, \dots, n\}$ be a finite set of experts. Expert $j \in N$ has a preference $\succ_j$ on $\mathcal{F}$. We use $\succ_0$ to denote the DM’s preference on $\mathcal{F}$. We suppose that, for all $i \in N$, expert i ’s preference is a Bewley preference with unique representation (u, C_i) . We thus assume in particular that there is no diversity of preferences over outcomes, which is a distinctive element of the theory of the aggregation of opinions, compared to the theory of the aggregation of preferences. We study how $\succ_0$ should depend on $(\succ_j)_{j \in N}$ and impose the two following conditions:

Axiom 9 (Pareto). For all $f, g \in F$, if $f \succ_i g$ for all $i \in N$, then $f \succ_0 g$.

Axiom 10 (Caution for incomparability). For all $f \in F$ and $x \in X$, if there exists $i \in N$ such that $f \bowtie_i x$, then $f \bowtie_0 x$.

The Pareto condition is the standard one. It asserts that the DM should follow the comparisons expressed by experts when they are *unanimous*: if all experts prefer act f to

⁴¹In particular, given this set of scenarios, the condition under which they have sufficient conviction that an option is better than an other one is stronger than if they had a concordant hope-and-prepare preference relation.

act g , the DM should also favor f over g . Caution for incomparability focuses on situations without clear comparisons. Based on the idea that the reason why the DM wants to rely on the opinions of experts to take decisions is that the issue at hand is *crucial* to her, it states that if some experts struggle to compare act f to a constant act x , while comparisons involving a constant act should be simpler, the DM should treat these acts as incomparable.

Interestingly, the result below demonstrates that these two axioms imply that the rule according to which the DM builds the relevant set of scenarios, on the basis of the beliefs of experts, is the same when the DM adopts a Bewley criterion as when she adopts a concordant hope-and-prepare one: the set of plausible scenarios considered by the DM is the same. The relative merits of the two criteria then depend on how the DM values conviction compared to decisiveness. If the DM wants to be more decisive, she should adopt the hope-and-prepare criterion. Conversely, if the DM prioritizes having stronger conviction in her decisions, the Bewley criterion, as considered by [Danan et al. \(2016\)](#), would be more suitable.⁴²

For all $P \subseteq \Delta$, we use $\text{co}(P)$ to denote the convex hull of P .

Proposition 3. *Suppose for all $i \in N$, $\succ_i$ is a Bewley preference with unique representation (u, C_i) , and either of the following holds:*

- $\succ_0$ is a hope-and-prepare preference with unique representation (u, C_0, C_0) , or
- $\succ_0$ is a Bewley preference with unique representation (u, C_0) .

Then,

- (i) *Pareto is satisfied if and only if $C_0 \subseteq \text{co}(\bigcup_{i=1}^n C_i)$.*
- (ii) *Caution for incomparability is satisfied if and only if $\text{co}(\bigcup_{i=1}^n C_i) \subseteq C_0$.*

In particular, when both conditions are met, $C_0 = \text{co}(\bigcup_{i=1}^n C_i)$.

7 Conclusion

We provided a new perspective on the analysis of incomplete preferences under uncertainty by introducing and characterizing a new decision criterion involving multiple priors. It is based on a requirement of unanimity between an optimistic and a pessimistic evaluation reflecting the behavior of a DM who *hopes for the best while she also prepares for the worst*.

⁴²This result shows in particular that a concordant hope-and-prepare preference relation, which, given a set of scenarios, is more complete than a Bewley preference relation, is still compatible with the limitation of comparability embedded in our caution axiom.

When both of these evaluations are computed according to the same set of scenarios, hope-and-prepare preferences compare ranges of expected utility according to a strict version of the well known and commonly used strong set order.

Comparing hope-and-prepare preferences to the two closest incomplete criteria proposed in this framework —Bewley and twofold preferences— we argued, and made visible in our axiomatization, that the trade-off between decisiveness and conviction is addressed in a way that is more favorable to decisiveness.

We showed that an *invariant biseparable completion* of a hope-and-prepare preference relation necessarily admits a *unique asymmetric α -maxmin* representation.

Finally, we proved that the rule according to which a DM builds a relevant set of scenarios, when she must aggregate the opinions of experts, is, under two conditions imposed on her decisions, the same whether she follows a Bewley criterion or a concordant hope-and-prepare one: then, we interpreted the adoption of one or the other criterion as a choice between the strength of her conviction and her ability to take decisions.

Appendix

A Strategic environments

We pointed out the conceptual relation between the way strategies are compared in [Troyan and Morrill \(2020\)](#) and the way acts are compared according to a hope-and-prepare preference relation. We formalize briefly this connection, restricting attention to direct mechanisms with a finite state of types, as in [Troyan and Morrill \(2020\)](#).

Two preliminary observations should be made. First, unlike [Li \(2017\)](#), [Troyan and Morrill \(2020\)](#) focus on a finite set of outcomes, while we assume that the set of outcomes is a convex subset of a Euclidean space. Secondly, the DM with a hope-and-prepare preference relation has a non-constant affine utility function on outcomes, while, in principle, the logic behind non-obvious domination should apply to more general preferences defined on a convex set. Of course, the same difference about admissible preferences holds between the criteria characterized by [Echenique et al. \(2022\)](#) and the notion of obvious domination of [Li \(2017\)](#).

Consider a finite set of n agents, $I = \{i_1, \dots, i_n\}$. The set of outcomes X is a convex subset of a Euclidean space. Agents have preferences over outcomes which are indexed by types $\theta_i \in \Theta_i$, where Θ_i is the finite set of possible types for agent i . The function $u_i(\cdot; \theta_i)$ denotes agent i 's utility function when her type is θ_i ; it is a non-constant affine mapping from X to $\mathbb{R}$. Let $\Theta_{-i} = \times_{j \in I \setminus \{i\}} \Theta_j$, of generic element θ_{-i} . Letting $\Theta_I = \times_{i \in I} \Theta_i$, a direct mechanism is a mapping $\phi : \Theta_I \rightarrow X$.

Let $i \in N$ be of type $\theta_i \in \Theta_i$. Let $S = \Theta_{-i}$. Given the mechanism ϕ , reporting $\theta'_i \in \Theta_i$ yields an act

$$\begin{aligned} f^{\theta'_i} : S &\rightarrow X \\ \theta_{-i} &\mapsto \phi(\theta'_i; \theta_{-i}). \end{aligned}$$

Under the proviso above, we say that the strategy consisting in reporting θ'_i in mechanism ϕ dominates the one consisting in reporting θ_i in the sense of [Troyan and Morrill \(2020\)](#) when

$$\begin{cases} \min_{\theta_{-i} \in \Theta_{-i}} u_i(\phi(\theta'_i, \theta_{-i}); \theta_i) > \min_{\theta_{-i} \in \Theta_{-i}} u_i(\phi(\theta_i, \theta_{-i}); \theta_i) \\ \max_{\theta_{-i} \in \Theta_{-i}} u_i(\phi(\theta'_i, \theta_{-i}); \theta_i) > \max_{\theta_{-i} \in \Theta_{-i}} u_i(\phi(\theta_i, \theta_{-i}); \theta_i) \end{cases},$$

which holds if and only if

$$\begin{cases} \min_{p \in C} \int u_i(f^{\theta'_i}; \theta_i) dp > \min_{p \in C} \int u_i(f^{\theta_i}; \theta_i) dp \\ \max_{p \in C} \int u_i(f^{\theta'_i}; \theta_i) dp > \max_{p \in C} \int u_i(f^{\theta_i}; \theta_i) dp \end{cases},$$

where C is the set of finitely additive probability measures on $(S, 2^S)$.

B Degree of conservatism and ambiguity attitudes of hope-and-prepare preferences

Let $\succsim_1$ and $\succsim_2$ be two hope-and-prepare preferences with representations (u, C_1, D_1) and (u, C_2, D_2) , respectively. We identify conditions under which $C_2 \subseteq C_1$ and $D_2 \subseteq D_1$, but $\succsim_1$ is not more conservative than $\succsim_2$. Consider $f, g \in \mathcal{F}$ such that there are $s_1, s_2 \in S$ satisfying

$$\begin{cases} u(f(s_1)) > u(g(s_1)) \\ u(f(s_2)) < u(g(s_2)) \\ u(f(s)) = u(g(s)) \text{ for all } s \neq s_1, s_2. \end{cases}$$

Assume that the utility function u is such that $u(f(s_1)) = u(g(s_2)) = 1$ and $u(f(s_2)) = u(g(s_1)) = 0$. Define p_1 , p_2 and p_3 as follows

$$\begin{cases} p_1(s_1) = \frac{1}{3}, p_1(s_2) = \frac{2}{3}, p_1(s) = 0 \forall s \neq s_1, s_2 \\ p_2(s_1) = 1, p_2(s_2) = 0, p_2(s) = 0 \forall s \neq s_1, s_2 \\ p_3(s_1) = \frac{2}{5}, p_3(s_2) = \frac{3}{5}, p_3(s) = 0 \forall s \neq s_1, s_2. \end{cases}$$

Now let $C_1 = C_2 = \{p_2\}$, $D_1 = \text{co}(\{p_1, p_2\})$ and $D_2 = \text{co}(\{p_1, p_3\})$, where co denotes the operator that associates with any subset of Δ its convex hull in Δ . One readily obtains:

$$\begin{aligned} \min_{p \in C_1} \int u(f) dp &= 1 > 0 = \min_{p \in C_1} \int u(g) dp, \\ \max_{p \in D_1} \int u(f) dp &= 1 > \frac{2}{3} = \max_{p \in D_1} \int u(g) dp, \\ \max_{p \in D_2} \int u(f) dp &= \frac{2}{5} < \frac{2}{3} = \max_{p \in D_2} \int u(g) dp, \end{aligned}$$

that is, $f \succ_1 g$ but $f \not\succ_2 g$.

C Proofs

C.1 Proof of Theorem 1

Only-if part. Assume that $\succ$ satisfies Axioms 1-5 and Axiom 6.

Consider the restriction of $\succ$ to X and define, for all $x, y \in X$, the binary relation $\gtrsim$ by $x \gtrsim y$ if and only if $y \not\succ x$. Clearly, $\gtrsim$ is complete and transitive on X and $\gtrsim$ is equivalent to $\sim$ on X . By Axiom 3, for all $x, y, z \in X$, $x \gtrsim y$ if and only if $\alpha x + (1 - \alpha)z \gtrsim \alpha y + (1 - \alpha)z$. Then, Axiom 2 implies that there exists an affine function $u : X \rightarrow \mathbb{R}$, unique up to affine transformation, such that $x \gtrsim y$ if and only if $u(x) \geq u(y)$. Also, u is non-constant as $\succ$ is non-trivial.

Let us now introduce intermediary results on which our proof is based.

Lemma 1. *Let $f, g \in \mathcal{F}$ be such that $f(s) \gtrsim g(s)$ for all $s \in S$. For all $h \in \mathcal{F}$, if $g \succ h$, then $f \succ h$, and if $h \succ f$, then $h \succ g$.*

Proof. We prove the first claim, the second being analogous. Let $h \in \mathcal{F}$ be such that $g \succ h$. As $\succ$ is non-trivial, there are $x, y \in X$ such that $x \succ y$. Let $\alpha \in (0, 1)$. By Axiom 3, $\alpha g + (1 - \alpha)x \succ \alpha h + (1 - \alpha)x$. By Axiom 2, there exists $\lambda \in (0, 1)$ such that $\alpha g + (1 - \alpha)(\lambda x + (1 - \lambda)y) \succ \alpha h + (1 - \alpha)x$. Since $u(f(s)) \geq u(g(s))$ for all $s \in S$ and

$u(x) > u(\lambda x + (1 - \lambda)y)$, one has $[\alpha f + (1 - \alpha)x](s) \succ [\alpha g + (1 - \alpha)(\lambda x + (1 - \lambda)y)](s)$ for all $s \in S$. By Axiom 5, $\alpha f + (1 - \alpha)x \succ \alpha g + (1 - \alpha)(\lambda x + (1 - \lambda)y)$. By transitivity, $\alpha f + (1 - \alpha)x \succ \alpha h + (1 - \alpha)x$ and Axiom 3 yields $f \succ h$. $\square$

Let us state an immediate consequence of Lemma 1 for constant acts.

Let $f \in \mathcal{F}$ and let $x, y \in X$ be such that $x \gtrsim y$. If $f \succ x$, then $f \succ y$. If $y \succ f$, then $x \succ f$. (2)

Lemma 2. *Let $f \in \mathcal{F}$ and $x, y \in X$:*

(i) *if $f \bowtie x$ and $f \succ y$, then $x \succ y$;*

(ii) *if $f \bowtie x$ and $y \succ f$, then $y \succ x$.*

Proof. We prove (i), the proof of (ii) being analogous. Assume $x \not\succ y$, by contradiction. Having $y \succ x$ would contradict $f \bowtie x$ by the transitivity of $\succ$. Thus, $y \sim x$, so that $y(s) \gtrsim x(s)$ for all $s \in S$. By Lemma 1, $f \succ y$ implies $f \succ x$, which contradicts $f \bowtie x$. $\square$

Lemma 3. *Let $f \in \mathcal{F}$ and $x \in X$ be such that $f \succ x$. Then, there exists $y \in X$ such that $f \succ y \succ x$.*

Proof. Let $z \in X$ be such that $z \gtrsim f(s)$ for all $s \in S$. By Lemma 1, $z \succ x$. By Axiom 2 and the non-constant affine utility representation of $\succ$ on X , there exists $\alpha \in (0, 1)$ such that $f \succ \alpha x + (1 - \alpha)z \succ x$. $\square$

Lemma 4. *For all $f \in \mathcal{F}$, the set $\{x \in X : x \bowtie f\}$ is non-empty.*

Proof. Let $f \in \mathcal{F}$. By definition of a simple act, there are x^* and x_* in X such that $x^* \gtrsim f(s) \gtrsim x_*$ for all $s \in S$. By Lemma 1, $x_* \not\succ f$ and $f \not\succ x^*$. The sets $\{\alpha \in [0, 1] : f \not\succ \alpha x^* + (1 - \alpha)x_*\}$ and $\{\alpha \in [0, 1] : \alpha x^* + (1 - \alpha)x_* \not\succ f\}$ are non-empty and closed by Axiom 2. Their union is $[0, 1]$, and by connectedness their intersection is non-empty. $\square$

The pessimistic weak order. We define the relation $\gtrsim_p$ on $\mathcal{F}$ by

$$f \gtrsim_p g \iff \text{for all } x \in X, g \succ x \text{ implies } f \succ x.$$

We denote by $\succ_p$ and $\sim_p$ the asymmetric and symmetric parts of $\gtrsim_p$, respectively. Note that $\gtrsim_p$ is transitive, and that for any two constant acts $x, y \in X$, $x \gtrsim_p y$ if and only if $x \gtrsim y$. We show that $\gtrsim_p$ is a continuous and monotone weak order satisfying certainty independence and displaying aversion to ambiguity.

Completeness. Let $f, g \in \mathcal{F}$ be such that $g \not\gtrsim_p f$. That is, there exists $x \in X$ such that $f \succ x$ but $g \not\succ x$. If there is no $y \in X$ such that $g \succ y$, then $f \gtrsim_p$ is trivially satisfied. Now, let $y \in X$ be such that $g \succ y$. Then, one has $x \gtrsim y$.⁴³ By (2), this means that $f \succ y$. We have thus proved $f \gtrsim_p g$.

Continuity. Let $f, g, h \in \mathcal{F}$. Consider the set $\{\alpha \in [0, 1] : \alpha f + (1 - \alpha)g \succ_p h\}$. By completeness of $\gtrsim_p$, $\alpha f + (1 - \alpha)g \succ_p h$ if and only if there exists $x \in X$ such that $\alpha f + (1 - \alpha)g \succ x$ and $h \not\succ x$. Therefore, $\{\alpha \in [0, 1] : \alpha f + (1 - \alpha)g \succ_p h\} = \bigcup_{x: h \not\succ x} \{\alpha : \alpha f + (1 - \alpha)g \succ x\}$. Each set in the union is open by Axiom 2; thus their union is open.

Consider the set $\{\alpha \in [0, 1] : h \succ_p \alpha f + (1 - \alpha)g\}$. Suppose $h \succ_p \alpha_0 f + (1 - \alpha_0)g$ for some $\alpha_0 \in [0, 1]$, so that there exists $x \in X$ such that $h \succ x$ and $\alpha_0 f + (1 - \alpha_0)g \not\succ x$. By Lemma 3, there exists $\hat{x} \in X$ such that $h \succ \hat{x} \succ x$. For α close enough to α_0 , $\frac{1}{2}[\alpha_0 f(s) + (1 - \alpha_0)g(s)] + \frac{1}{2}\hat{x} \succ \frac{1}{2}[\alpha f(s) + (1 - \alpha)g(s)] + \frac{1}{2}x$ for all $s \in S$. Assume $\alpha f + (1 - \alpha)g \succ \hat{x}$, by contradiction. Axiom 3 yields $\frac{1}{2}[\alpha f + (1 - \alpha)g] + \frac{1}{2}x \succ \frac{1}{2}\hat{x} + \frac{1}{2}x$. Combined with the state-wise dominance above, Axiom 5, and Axiom 3, this implies that $\alpha_0 f + (1 - \alpha_0)g \succ x$, which is a contradiction. Thus, $\alpha f + (1 - \alpha)g \not\succ \hat{x}$, and hence $h \succ_p \alpha f + (1 - \alpha)g$. We have proved that $\{\alpha \in [0, 1] : h \succ_p \alpha f + (1 - \alpha)g\}$ is open.

Monotonicity. Let $f, g \in \mathcal{F}$ such that $f(s) \gtrsim g(s)$ for all $s \in S$. For any $x \in X$ with $g \succ x$, Lemma 1 gives $f \succ x$. Thus $f \gtrsim_p g$.

Certainty independence. As a recall, we will show that for all $f, g \in \mathcal{F}$, $z \in X$, and $\alpha \in (0, 1)$, $f \gtrsim_p g$ if and only if $\alpha f + (1 - \alpha)z \gtrsim_p \alpha g + (1 - \alpha)z$.

Suppose $\alpha f + (1 - \alpha)z \gtrsim_p \alpha g + (1 - \alpha)z$. Let $x \in X$ be such that $g \succ x$. By Axiom 3, $\alpha g + (1 - \alpha)z \succ \alpha x + (1 - \alpha)z$. Then $\alpha f + (1 - \alpha)z \succ \alpha x + (1 - \alpha)z$, and Axiom 3 gives $f \succ x$. Thus, $f \gtrsim_p g$.

Suppose $f \gtrsim_p g$. Let $x \in X$ be such that $\alpha g + (1 - \alpha)z \succ x$. We must show $\alpha f + (1 - \alpha)z \succ x$. Let $x^* \in X$ be such that $x^* \gtrsim g(s)$ for all $s \in S$. By Lemma 1, $\alpha x^* + (1 - \alpha)z \succ x$. First, assume that there exists $x_* \in X$ such that $x \gtrsim \alpha x_* + (1 - \alpha)z$. By Axiom 2, there exists $\lambda \in [0, 1]$ such that $\alpha(\lambda x^* + (1 - \lambda)x_*) + (1 - \alpha)z \sim x$. Then the fact that $\alpha g + (1 - \alpha)z \succ x$ and (2) yield $\alpha g + (1 - \alpha)z \succ \alpha(\lambda x^* + (1 - \lambda)x_*) + (1 - \alpha)z$. By Axiom 3, $g \succ \lambda x^* + (1 - \lambda)x_*$. Since $f \gtrsim_p g$, one has $f \succ \lambda x^* + (1 - \lambda)x_*$. Applying Axiom 3 again, $\alpha f + (1 - \alpha)z \succ \alpha(\lambda x^* + (1 - \lambda)x_*) + (1 - \alpha)z$, and, by (2), we conclude that $\alpha f + (1 - \alpha)z \succ x$.

Now, assume that for all $y \in X$ $\alpha y + (1 - \alpha)z \succ x$. This comparison holds in particular for y_* such that, for all $s \in S$, $f(s) \gtrsim y_*$. Then, $[\alpha f + (1 - \alpha)z](s) \gtrsim [\alpha y_* + (1 - \alpha)z](s)$ for all $s \in S$, and we conclude, by Lemma 1, that $\alpha f + (1 - \alpha)z \succ x$.

Aversion to ambiguity. Let $f, g \in \mathcal{F}$ such that $f \sim_p g$. Then $\{x \in X : f \succ x\} = \{x \in X :$

⁴³Assume by contradiction that $y \succ x$. By Lemma 1, $g \succ x$, which is a contradiction. Thus, $x \gtrsim y$.

$g \succ x\}$. Let $x \in X$ such that $f \succ x$. Then $g \succ x$, and Axiom 4 gives $\alpha f + (1 - \alpha)g \succ x$. Thus $\alpha f + (1 - \alpha)g \succ_p f$.

The optimistic weak order. We define the relation $\succsim_o$ on $\mathcal{F}$ by

$$f \succsim_o g \iff \text{for all } x \in X, x \succ f \text{ implies } x \succ g.$$

By arguments that are completely symmetric to those given for $\succsim_p$, one shows that $\succsim_o$ is a continuous and monotone weak order satisfying certainty independence and displaying preference for ambiguity.⁴⁴

Representations and remaining claims. It is well-known since Gilboa and Schmeidler (1989) that a weak order on $\mathcal{F}$ satisfying the properties established above can be represented by $f \mapsto \min_{p \in C} \int u_p(f) dp$ if it displays ambiguity aversion, such as $\succsim_p$, and by $f \mapsto \max_{p \in D} \int u_o(f) dp$ if it displays love for ambiguity, such as $\succsim_o$, where C and D are unique non-empty convex compact subsets of Δ , u_p and u_o are two affine functions on X , unique up to positive affine transformation. Since $\succsim_p$ and $\succsim_o$ coincide with $\succsim$ on X , one may assume $u_p = u_o = u$. It remains to show that $f \succ g$ if and only if $f \succ_p g$ and $f \succ_o g$, and that $C \cap D \neq \emptyset$.

Claim 1. For all $f, g \in \mathcal{X}$, $f \succ g \iff f \succ_p g$ and $f \succ_o g$.

Let $f, g \in \mathcal{F}$ be such that $f \succ_p g$ and $f \succ_o g$. Since $g \not\succsim_p f$, there exists $x \in X$ such that $f \succ x$ and $g \not\succ x$: either $x \succ g$, giving $f \succ g$ by transitivity, or $x \bowtie g$. Similarly, since $g \not\succsim_o f$, there exists $y \in X$ such that $y \succ g$ and $y \not\succ f$: either $f \succ y$, giving $f \succ g$ by transitivity, or $y \bowtie f$. If both $x \bowtie g$ and $y \bowtie f$, then Axiom 6 implies $f \succ g$.

Let $f, g \in \mathcal{F}$ such that $f \succ g$. Transitivity implies that for all $x \in X$, $g \succ x$ implies $f \succ x$ and $x \succ f$ implies $x \succ g$, so that $f \succsim_p g$ and $f \succsim_o g$. We will show that these comparisons must be strict. Let $x, y \in X$ be such that $x \succ y$ and $\alpha \in (0, 1)$. By Axiom 3, $\alpha f + (1 - \alpha)x \succ \alpha g + (1 - \alpha)x$. By Axiom 2, there exists $\lambda \in (0, 1)$ such that $\alpha f + (1 - \alpha)(\lambda x + (1 - \lambda)y) \succ \alpha g + (1 - \alpha)x$. By transitivity, $\alpha f + (1 - \alpha)(\lambda x + (1 - \lambda)y) \succsim_p \alpha g + (1 - \alpha)x$ and $\alpha f + (1 - \alpha)(\lambda x + (1 - \lambda)y) \succsim_o \alpha g + (1 - \alpha)x$. Using the representation of $\succsim_p$:

$$\alpha \min_{p \in C} \int u(f) dp + (1 - \alpha)u(\lambda x + (1 - \lambda)y) \geq \alpha \min_{p \in C} \int u(g) dp + (1 - \alpha)u(x),$$

and thus $\min_{p \in C} \int u(f) dp - \min_{p \in C} \int u(g) dp \geq \frac{(1 - \alpha)}{\alpha} (1 - \lambda)(u(x) - u(y)) > 0$, which gives $f \succ_p g$. An analogous computation yields $f \succ_o g$.

⁴⁴Alternatively, define $\succ^*$ on $\mathcal{F}$ by $f \succ^* g$ if and only if $g \succ f$. One verifies that $\succ^*$ satisfies the same axioms as $\succ$. Then $\succsim_o$ coincides with the relation obtained by applying the analysis of $\succsim_p$ to $\succ^*$.

Finally, we show that C and D are non-disjoint.

Claim 2. $C \cap D \neq \emptyset$ if, and only if, $\min_{p \in C} \int u(f)dp \leq \max_{p \in D} \int u(f)dp$ for all $f \in \mathcal{F}$

We only prove the *if part*, the other direction being trivial. Suppose $C \cap D = \emptyset$. By the separating hyperplane theorem, there exists a bounded measurable function $\varphi : S \rightarrow \mathbb{R}$ such that $\min_{p \in C} \int \varphi dp > \max_{p \in D} \int \varphi dp$. Approximating φ by a sequence of simple functions $\{\varphi_n\}$ in supnorm and rescaling, one obtains that there is $f \in \mathcal{F}$ such that $\min_{p \in C} \int u(f)dp > \max_{p \in D} \int u(f)dp$.

It remains to show that $\min_{p \in C} \int u(f)dp \leq \max_{p \in D} \int u(f)dp$ for all $f \in \mathcal{F}$. It suffices to show that for all $f \in \mathcal{F}$, $f \succ_p x$ implies $f \succ_o x$ for all $f \in \mathcal{F}$ and $x \in X$.

So let $f \in \mathcal{F}$ and $x \in X$ be such that $f \succ_p x$. One has $f \succ x$. Indeed, $x \not\prec_p f$ implies that there is $y \in X$ such that $f \succ y$ and $x \not\prec y$. Since x and y are constant acts, $x \not\prec y$ gives $y \succ x$, and (2) yields $f \succ x$. Now, let $z \in X$ such that $z \bowtie f$ (Lemma 4). By Lemma 2 (i), $z \succ x$. Since $z \bowtie f$ and $z \succ x$, on the one hand, $w \succ f$ implies $w \succ z$ by Lemma 2 (ii), and thus $w \succ x$ by transitivity, which gives $f \succ_o x$. On the other hand, $z \succ x$ but $z \not\prec f$ (as $z \bowtie f$), so $x \not\prec_o f$. Thus $f \succ_o x$.

We have thus proved that C and D are non-disjoint.

If part. Assume that $\succ$ admits a hope-and-prepare representation. One can readily check that Axioms 1 to 5 are satisfied.

For all $f \in \mathcal{F}$, denote $\bar{p}_f \in \arg \max_{p \in D} \int u(f)dp$ and $\underline{p}_f \in \arg \min_{p \in C} \int u(f)dp$. Define also the constant acts $\bar{f} = \int f d\bar{p}_f$ and $\underline{f} = \int f d\underline{p}_f$. Clearly, $f \bowtie \bar{f}$ and $f \bowtie \underline{f}$; moreover,

$$\begin{cases} f \succ x & \iff u(\underline{f}) > u(x), \\ x \succ f & \iff u(x) > u(\bar{f}), \\ f \bowtie x & \iff u(\bar{f}) \geq u(x) \geq u(\underline{f}) \end{cases} . \quad (3)$$

Axiom 6 easily obtains from the comparisons in (3). Indeed, let $f, g \in \mathcal{F}$ and $x, y \in X$ be such that $f \bowtie x$, $g \bowtie y$, $x \succ g$, and $f \succ y$. Using (3), one gets $u(\bar{f}) \geq u(x) > u(\bar{g})$ and $u(\underline{f}) > u(y) \geq u(\underline{g})$, that is $f \succ g$, by definition of a hope-and-prepare preference.

C.2 Proof of Theorem 2

We prove (i); the proof of (ii) is symmetric.

Suppose $D \subseteq C$. Let $f \in \mathcal{F}$, $x \in X$ be such that $f \succ x$, $h \in \mathcal{F}$, and $\alpha \in (0, 1]$.

Consider first the maxmin evaluation:

$$\min_{p \in C} \int u(\alpha f + (1 - \alpha)h) dp \geq \alpha \min_{p \in C} \int u(f) dp + (1 - \alpha) \min_{p \in C} \int u(h) dp.$$

Since $f \succ x$ implies $\min_{p \in C} \int u(f) dp > u(x)$, the right-hand side is strictly greater than $\alpha u(x) + (1 - \alpha) \min_{p \in C} \int u(h) dp = \min_{p \in C} \int u(\alpha x + (1 - \alpha)h) dp$.

Consider now the maxmax evaluation. Let $q \in \arg \max_{p \in D} \int u(h) dp$. Since $D \subseteq C$, one has $q \in C$, so that $\int u(f) dq \geq \min_{p \in C} \int u(f) dp > u(x)$. As a consequence,

$$\begin{aligned} \max_{p \in D} \int u(\alpha f + (1 - \alpha)h) dp &\geq \alpha \int u(f) dq + (1 - \alpha) \max_{p \in D} \int u(h) dp \\ &> \alpha u(x) + (1 - \alpha) \max_{p \in D} \int u(h) dp, \end{aligned}$$

where the last terms is equal to $\max_{p \in D} \int u(\alpha x + (1 - \alpha)h) dp$. Thus, $\alpha f + (1 - \alpha)h \succ \alpha x + (1 - \alpha)h$.

Conversely, suppose $D \not\subseteq C$. There exists $p^* \in D \setminus C$. By the separating hyperplane theorem, there exist $f \in \mathcal{F}$ and $x \in X$ such that

$$\int u(f) dp^* < u(x) < \min_{p \in C} \int u(f) dp.$$

Since $C \cap D \neq \emptyset$, one has $\max_{p \in D} \int u(f) dp \geq \min_{p \in C} \int u(f) dp > u(x)$, so that $f \succ x$.

Now, let $h \in \mathcal{F}$ and $\alpha \in (0, 1)$ such that $\alpha f + (1 - \alpha)h$ is a constant act.⁴⁵ The fact that $\alpha f + (1 - \alpha)h$ is a constant act implies

$$\max_{p \in D} \int u(\alpha f + (1 - \alpha)h) dp = \alpha \int u(f) dp^* + (1 - \alpha) \int u(h) dp^*.$$

Since $\int u(f) dp^* < u(x)$, one has

$$\begin{aligned} \alpha \int u(f) dp^* + (1 - \alpha) \int u(h) dp^* &< \alpha u(x) + (1 - \alpha) \int u(h) dp^* \\ &\leq \max_{p \in D} \int u(\alpha x + (1 - \alpha)h) dp. \end{aligned}$$

Hence $\alpha f + (1 - \alpha)h \not\succeq \alpha x + (1 - \alpha)h$, that is, Axiom 7 does not hold.

⁴⁵Such α and h always exist.

C.3 Proof of Proposition 1

i) Let $\succ_{HP}$ be a hope-and-prepare preference with unique representation (u, C_{HP}, D_{HP}) , and let $\succ_B$ be Bewley preference with unique representation (u, C_B) .

First, suppose that $C_{HP} \cup D_{HP} \subseteq C_B$. If $f \succ_B g$, then for all $p \in C_{HP} \cup D_{HP}$,

$$\int u(f)dp > \int u(g)dp,$$

which implies, as C_{HP} and D_{HP} are not disjoint,

$$\begin{cases} \min_{p \in C_{HP}} \int u(f)dp > \min_{p \in C_{HP}} \int u(g)dp, \\ \max_{p \in D_{HP}} \int u(f)dp > \max_{p \in D_{HP}} \int u(g)dp. \end{cases}$$

Therefore, $f \succ_{HP} g$. Thus, $\succ_B$ is more conservative than $\succ_{HP}$.

Conversely, suppose $\succ_B$ is more conservative than $\succ_{HP}$ and suppose, by contradiction, that there exists $p^* \in C_{HP} \setminus C_B$. By the separation argument we already used in the Conclusion step of the proof of Theorem 1, there are $f \in \mathcal{F}$ and $x \in X$ such that

$$\int u(f)dp^* > u(x) > \max_{p \in C_B} \int u(f)dp.$$

It follows that $x \succ_B f$ but $x \not\succ_{HP} f$, a contradiction. Similarly, suppose there exists $p^* \in D_{HP} \setminus C_B$. Then there are $f \in \mathcal{F}$ and $x \in X$ such that

$$\min_{p \in C_B} \int u(f)dp > u(x) > \int u(f)dp^*.$$

In this case, we have $f \succ_B x$ but $f \not\succ_{HP} x$, an other contradiction. Therefore, $C_{HP} \cup D_{HP} \subseteq C_B$.

ii) Let $\succ_{HP}$ be a hope-and-prepare preference with unique representation (u, C_{HP}, D_{HP}) , and $\succ_T$ be a twofold multiprior preference with unique representation (u, C_T, D_T) .

First, suppose that $C_{HP} \subseteq C_T$ and $D_{HP} \subseteq D_T$. Since $D_T \cap C_T \neq \emptyset$ and $D_{HP} \cap C_{HP} \neq \emptyset$, $C_{HP} \cap D_T \neq \emptyset$ and $D_{HP} \cap C_T \neq \emptyset$. If $f \succ_T g$, then

$$\min_{p \in C_T} \int u(f)dp > \max_{p \in D_T} \int u(g)dp,$$

which implies

$$\begin{aligned} \min_{p \in C_{HP}} \int u(f) dp &\geq \min_{p \in C_T} \int u(f) dp > \max_{p \in D_T} \int u(g) dp \geq \max_{p \in D_{HP}} \int u(g) dp, \\ \max_{p \in D_{HP}} \int u(f) dp &\geq \min_{p \in C_T} \int u(f) dp > \max_{p \in D_T} \int u(g) dp \geq \max_{p \in D_{HP}} \int u(g) dp. \end{aligned}$$

Since $D_{HP} \cap C_{HP} \neq \emptyset$, one gets

$$\max_{p \in D_{HP}} \int u(f) dp \geq \min_{p \in C_{HP}} \int u(f) dp > \max_{p \in D_{HP}} \int u(g) dp \geq \min_{p \in C_{HP}} \int u(g) dp.$$

Therefore, $f \succ_{HP} g$. Thus, $\succ_T$ is more conservative than $\succ_{HP}$.

Conversely, suppose $\succ_T$ is more conservative than $\succ_{HP}$ and suppose, by contradiction, that there exists $p^* \in C_{HP} \setminus C_T$. There are $f \in \mathcal{F}$ and $x \in X$ such that

$$\min_{p \in C_T} \int u(f) dp > u(x) > \int u(f) dp^*,$$

from which it follows that $f \succ_T x$ but $f \not\succ_{HP} x$, a contradiction. To prove that $D_{HP} \subseteq D_T$, suppose there exists $p^* \in D_{HP} \setminus D_T$. There are $f \in \mathcal{F}$ and $x \in X$ such that

$$\int u(f) dp^* > u(x) > \max_{p \in D_T} \int u(f) dp.$$

In this case, $x \succ_T f$ but $x \not\succ_{HP} f$, an other contradiction.

C.4 Proof of Proposition 2

Clearly, for each $i \in \{1, 2\}$, and all $x \in X$, $f \succ_i x$ if and only if $f \succ_{ip} x$, where $\succ_{ip}$ is the pessimistic relation defined, as in the proof of Theorem 1, by $f \succ_{ip} g$ if and only if $f \succ_i y$ and $y \bowtie_i g$ for some $y \in X$. Thus, $\succ_1$ is more ambiguity averse than $\succ_2$ if and only if $\succ_{1p}$ is more ambiguity averse than $\succ_{2p}$. When proving Theorem 1, we have shown that $\succ_{ip}$ is represented by a maxmin expected utility functional; therefore, $\succ_{1p}$ is more ambiguity averse than $\succ_{2p}$ if and only if $C_2 \subseteq C_1$.

Similarly, for each $i \in \{1, 2\}$, and all $x \in X$, $x \succ_i f$ if and only if $x \succ_{io} f$, where $\succ_{io}$ is the optimistic relation defined, as in the proof of Theorem 1, by $f \succ_{io} g$ if and only if $f \bowtie_i y$ and $y \succ_i g$ for some $y \in X$. As we have proved that $\succ_{io}$ admits a maxmax expected utility representation, one obtains that $\succ_1$ is more ambiguity loving than $\succ_2$ if and only if $D_2 \subseteq D_1$.

C.5 Proof of Theorem 3

We prove that (i) implies (ii); the converse is routine.

The proof relies on two lemmas. The first provides a representation for invariant biseparable preferences.

Lemma 5. *An asymmetric, complete, and negatively transitive binary relation $\succ$ on $\mathcal{F}$ satisfies Axioms 2, 3, and 5 if and only if there exist a monotonic, constant-linear functional $I : B_0(\Sigma) \rightarrow \mathbb{R}$ and a non-constant affine function $u : X \rightarrow \mathbb{R}$ such that, for all $f, g \in \mathcal{F}$,*

$$f \succ g \iff I(u(f)) > I(u(g)).$$

Moreover, I is unique and u is unique up to positive affine transformation.

Proof. Define $\succeq$ by $f \succeq g$ if and only if $g \not\succ f$. Then $\succeq$ is a complete and transitive weak order. Continuity of $\succeq$ follows from Axiom 2. Monotonicity of $\succeq$ follows from Lemma 1: if $f(s) \succeq g(s)$ for all $s \in S$ and $g \succ f$, then $g \succ g$ by the lemma, contradicting asymmetry. Certainty independence carries over directly from Axiom 3. The result then follows from Lemma 1 in Ghirardato et al. (2004). $\square$

The second lemma characterizes when a monotonic constant-linear functional can be decomposed as a convex combination of two others. Its proof closely follows that of Lemma A.3 in Frick et al. (2022).

Lemma 6. *Let $I, I', I'' : B_0(\Sigma) \rightarrow \mathbb{R}$ be monotonic and constant-linear with $I' \leq I''$. Then the following are equivalent:*

- (i) *For all $\varphi, \psi \in B_0(\Sigma)$, $I'(\varphi) > I'(\psi)$ and $I''(\varphi) > I''(\psi)$ implies $I(\varphi) > I(\psi)$.*
- (ii) *There exists $\alpha \in [0, 1]$ such that $I = \alpha I' + (1 - \alpha)I''$.*

Proof. That (ii) implies (i) is immediate. We prove the converse.

By (i), there exists an increasing function W such that $I(\varphi) = W(I'(\varphi), I''(\varphi))$ for all φ . If $I'(\varphi) = I''(\varphi) = k$, then for all $\varepsilon > 0$, monotonicity and constant-linearity of I' and I'' give $I'(k + \varepsilon) > I'(\varphi) > I'(k - \varepsilon)$ and similarly for I'' ; by (i), $k + \varepsilon > I(\varphi) > k - \varepsilon$, so $I(\varphi) = k$.

For φ with $I'(\varphi) < I''(\varphi)$, define $\alpha(\varphi) = (I(\varphi) - I''(\varphi))/(I'(\varphi) - I''(\varphi))$. It remains to show $\alpha(\varphi)$ is constant. Set $\psi = (\varphi - I''(\varphi))/(I''(\varphi) - I'(\varphi))$; then $I'(\psi) = -1$ and $I''(\psi) = 0$, so $\alpha(\varphi) = -I(\psi) = -W(-1, 0)$, which is independent of φ . Setting $\alpha = -W(-1, 0)$, one has $I = \alpha I' + (1 - \alpha)I''$.

To show $\alpha \in [0, 1]$: if $\alpha < 0$, then for φ with $I'(\varphi) < I''(\varphi)$, $I(\varphi) > I''(\varphi)$, and choosing $\varepsilon > 0$ with $I(\varphi) > I''(\varphi) + \varepsilon$, monotonicity and (i) give $I''(\varphi) + \varepsilon > I(\varphi)$, a contradiction. The argument for $\alpha \leq 1$ is symmetric. $\square$

We now prove the theorem. Assume $\succ$ is a hope-and-prepare preference with unique representation (u, C, D) , and $\succ^*$ is an invariant biseparable extension of $\succ$. By Lemma 5, there exist a monotonic constant-linear functional $I : B_0(\Sigma) \rightarrow \mathbb{R}$ and a non-constant affine function $u' : X \rightarrow \mathbb{R}$ such that $f \succ^* g \iff I(u'(f)) > I(u'(g))$, where I is unique and u' is unique up to positive affine transformation. Since $\succ^*$ extends $\succ$, the two preferences agree on constant acts, so u' is a positive affine transformation of u ; we may thus assume $u' = u$.

Define $I'(\varphi) = \min_{p \in C} \int \varphi dp$ and $I''(\varphi) = \max_{p \in D} \int \varphi dp$ for all $\varphi \in B_0(\Sigma)$. These are monotonic and constant-linear, and $I' \leq I''$ since $C \cap D \neq \emptyset$.

We verify condition (i) of Lemma 6. Suppose $I'(\varphi) > I'(\psi)$ and $I''(\varphi) > I''(\psi)$ for some $\varphi, \psi \in B_0(\Sigma)$. Since $u(X)$ is an interval, there exist $a > 0$ and $b \in \mathbb{R}$ such that $\varphi' = a\varphi + b$ and $\psi' = a\psi + b$ belong to $B_0(\Sigma, u(X))$, and hence $\varphi' = u(f)$ and $\psi' = u(g)$ for some $f, g \in \mathcal{F}$. By constant-linearity, $I'(\varphi') > I'(\psi')$ and $I''(\varphi') > I''(\psi')$, which gives $f \succ g$. Since $\succ^*$ extends $\succ$, $f \succ^* g$, so $I(u(f)) > I(u(g))$, and constant-linearity yields $I(\varphi) > I(\psi)$.

By Lemma 6, there exists $\alpha \in [0, 1]$ such that $I = \alpha I' + (1 - \alpha)I''$, giving

$$I(u(f)) = \alpha \min_{p \in C} \int u(f) dp + (1 - \alpha) \max_{p \in D} \int u(f) dp$$

for all $f \in \mathcal{F}$.

Finally, if $\succ$ is incomplete, there exists $f \in \mathcal{F}$ with $\min_{p \in C} \int u(f) dp < \max_{p \in D} \int u(f) dp$. For any $\alpha' \neq \alpha$, one would have $\alpha' I'(u(f)) + (1 - \alpha') I''(u(f)) \neq I(u(f))$, contradicting the uniqueness of I . Thus α is unique.

C.6 Proof of Proposition 3

Let $C = \text{co}(\bigcup_{i=1}^n C_i)$. Clearly, C is a compact convex set. Let $\succ$ be a Bewley preference relation on $\mathcal{F}$, with representation (u, C) .

First, assume that $\succ_0$ is a hope-and-prepare preference with unique representation (u, C_0, C_0) .

i) Suppose that Pareto holds. Clearly, for all $f, g \in \mathcal{F}$, $f \succ g$ implies $f \succ_0 g$. Thus, by definition, $\succ$ is more conservative than the social preference $\succ_0$, and, by Proposition 1, $C_0 \subseteq C$. The converse implication is immediate.

ii) Assume that caution for incomparability holds. Suppose, by contradiction, that there

is $p^* \in C \setminus C_0$. Then, there are $f \in \mathcal{F}$ and $x \in X$ such that

$$\int u(f)dp^* > u(x) > \max_{p \in C_0} \int u(f)dp.$$

By definition of C , there exist $(p_i)_{i \in N} \in \times_{i \in N} C_i$ and $(\lambda_i)_{i \in N} \in \mathbb{R}_+^n$ such that $\sum_{i=1}^n \lambda_i = 1$ and $p^* = \sum_{i=1}^n \lambda_i p_i$. Thus, there exists $j \in N$ such that $\int u(f)dp_j > u(x) > \max_{p \in C_0} \int u(f)dp$, which implies $\max_{p \in C_j} \int u(f)dp > u(x) > \max_{p \in C_0} \int u(f)dp$. As $x \succ_0 f$ —this is the right hand side of the inequality above—caution for incomparability implies that either $x \succ_j f$ or $f \succ_j x$. Since $\max_{p \in C_j} \int u(f)dp > u(x)$, one concludes that $f \succ_j x$, which implies $\min_{p \in C_j} \int u(f)dp > u(x)$. Let $\bar{x} \in X$ such that $f \bowtie_j \bar{x}$:

$$u(\bar{x}) \geq \min_{p \in C_j} \int u(f)dp > u(x) > \max_{p \in C_0} \int u(f)dp,$$

which implies $\bar{x} \succ_0 f$. This is in contradiction with caution for incomparability. Therefore, $C \subseteq C_0$.

We now prove the converse implication. Assume $C \subseteq C_0$. Let $f \in \mathcal{F}$ and $x \in X$ such that $f \bowtie_i x$ for some $i \in N$. This implies

$$\max_{p \in C_i} \int u(f)dp \geq u(x) \geq \min_{p \in C_i} \int u(f)dp.$$

Since $C_i \subseteq C$, $C_i \subseteq C_0$. Hence,

$$\max_{p \in C_0} \int u(f)dp \geq \max_{p \in C_i} \int u(f)dp \geq u(x) \geq \min_{p \in C_i} \int u(f)dp \geq \min_{p \in C_0} \int u(f)dp,$$

that is, $f \bowtie_0 x$.

Now, assume that $\succ_0$ is a Bewley preference with unique representation (u, C_0) . Statement *i*) is an immediate consequence of Theorem 2 in [Danan et al. \(2016\)](#). Statement *ii*) in this case is proved from the same argument as statement *ii*) in the previous case.

The if part is immediate. Suppose that caution for incomparability holds. Suppose, by contradiction, that there exists $p^* \in C \setminus C_0$. There are $f \in \mathcal{F}$ and $x \in X$ such that

$$\int u(f)dp^* > u(x) > \max_{p \in C_0} \int u(f)dp.$$

Then, there is $j \in N$ such that

$$\max_{p \in C_j} \int u(f)dp > u(x) > \max_{p \in C_0} \int u(f)dp.$$

The right hand side of this inequality implies that $x \succ_0 f$. From caution for incomparability, either $x \succ_j f$ or $f \succ_j x$. By the left hand side of the inequality above, $f \succ_j x$. Let $\bar{x} \in X$ such that $f \bowtie_j \bar{x}$. By caution for incomparability, $f \bowtie_0 \bar{x}$. However, it also holds that $u(\bar{x}) \geq \min_{p \in C_j} \int u(f) dp > u(x) > \max_{p \in C_0} \int u(f) dp$, which implies $\bar{x} \succ_0 f$; a contradiction. Therefore, $C \subseteq C_0$.

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